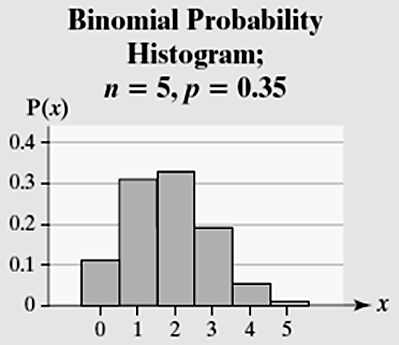
# Chapter 7 – The Normal Probability Distribution

## OUTLINE

1. Properties of the Normal Distribution
2. Applications of the Normal Distribution
3. Assessing Normality
4. The Normal Approximation to the Binomial Probability Distribution

## Putting It Together

In Chapter 6, we introduced discrete probability distributions. We computed probabilities using probability distribution functions. However, we could also determine the probability of any discrete random variable from its probability histogram. For example, the figure below shows the probability histogram for the binomial random variable *X* with *n* = 5 and *p* = 0.35.



From this probability histogram, we see that  Notice that the width of each rectangle in the probability histogram is 1. Since the area of a rectangle equals height times width, we can think of *P*(1) as the area of the rectangle corresponding to *x* = 1. Thinking of probability in this way makes the transition from computing discrete probabilities to finding continuous probabilities much easier.

In this chapter, we discuss two continuous distributions: the *uniform distribution* and the *normal distribution*. Most of the discussion will focus on the normal distribution, which has many applications.

## Section 7.1 Properties of the Normal Distribution

### Objectives

1. Use the Uniform Probability Distribution
2. Graph a Normal Curve
3. State the Properties of the Normal Curve
4. Explain the Role of Area in the Normal Density Function

#### Objective 1: Use the Uniform Probability Distribution

Objective 1, Page 1

We discuss a uniform distribution to see the relation between area and probability.

Objective 1, Page 2

**Example 1 *The Uniform Distribution***

Assume that United Parcel Service is supposed to deliver a package to your front door and the arrival time is somewhere between 10 AM and 11 AM. Let the random variable *X* represent the time from 10 AM when the delivery is supposed to take place.

The delivery could be at 10 AM (*x* = 0) or at 11 AM (*x* = 60), with all one-minute intervals of time between *x* = 0 and *x* = 60 equally likely. That is to say, your package is just as likely to arrive between 10:15 and 10:16 as it is to arrive between 10:40 and 10:41.

The random variable *X* can be any value in the interval from 0 to 60, that is,  Because any two intervals of equal length between 0 and 60, inclusive, are equally likely, the random variable *X* is said to follow a uniform probability distribution.

Objective 1, Page 3

 *Answer the following after watching the video.*

1. What two properties must a probability density function (pdf) satisfy?

Objective 1, Page.3(continued)

1. If the possible values of a uniform density function go from 0 through *n*, what is the height of the rectangle?
2. What does the area under the graph of a probability density function over an interval represent?

#### Objective 2: Graph a Normal Curve

Objective 2, Page 1

Not all continuous random variables follow a uniform distribution.

In Figure 1, as the class width of the histogram decreases, the histogram becomes closely approximated by the smooth red curve. For this reason, we can use the curve to *model* the probability distribution of this continuous random variable.

Objective 2, Page 2

1. What does it mean to say that a continuous random variable is normally distributed?

Objective 2, Page 3

*Answer the following while watching the video.*

1. What value of *x* is associated with the peak of a normal curve?
2. What values of *x* are associated with the inflection points of a normal curve?

Objective 2, Page 4

1. Sketch and label the graph from Figure 2.

Objective 2, Page 5

 *Answer the following after Activity 1: The Role of  and  in a Normal Curve.*

1. What happens to the graph as the mean increases? What happens to the graph as the mean decreases?
2. What happens to the graph as the standard deviation increases? What happens to the graph as the standard deviation decreases?

#### Objective 3: State the Properties of the Normal Curve

Objective 3, Page 1

1. State the seven properties of the normal curve.

#### Objective 4: Explain the Role of Area in the Normal Density Function

Objective 4, Page 1

 *Watch the video to see an example of a normally distributed random variable*.

The area under the normal curve can be used to model the probability histogram and the actual proportion in a given interval.

Objective 4, Page 2

1. Suppose that a random variable *X* is normally distributed with mean  and standard deviation . Give two representations for the area under the normal curve for any interval of values of the random variable *X*.

**Example 2 *Interpreting the Area Under a Normal Curve***

The serum total cholesterol for males 20 to 29 years old is approximately normally distributed with mean  and standard deviation , based on data obtained from the National Health and Nutrition Examination Survey.

1. Draw a normal curve with the parameters labeled.
2. An individual with total cholesterol greater than 200 is considered to have high cholesterol. Shade the region under the normal curve to the right of *x* = 200.
3. Suppose that the area under the normal curve to the right of *x* = 200 is 0.2903. (You will learn how to find this area in the next section.) Provide two interpretations of this result.

## Section 7.2 Applications of the Normal Distribution

### Objectives

1. Find and Interpret the Area under a Normal Curve
2. Find the Value of a Normal Random Variable

#### Objective 1: Find and Interpret the Area under a Normal Curve

Objective 1, Page 1

1. Suppose that the random variable *X* is normally distributed with mean  and standard deviation . Explain the distribution of the random variable . What is the name for the random variable *Z*?

Objective 1, Page 2

1. Explain how to find the area to the left of *x* for a normally distributed random variable *X*, using Table V.

Objective 1, Page 3

 *Answer the following after watching the video.*

1. Explain how to find the area to the right of *x* for a normally distributed random variable *X*, using Table V.

Objective 1, Page 5

**Example 1 *Finding and Interpreting Area Under a Normal Curve***

A pediatrician obtains the heights of her 200 three-year-old female patients. The heights are approximately normally distributed, with mean 38.72 inches and standard deviation 3.17 inches. Use the normal model to determine the proportion of the 3-year-old females who have a height less than 35 inches.

Objective 1, Page 6

Note that the proportion of 3-year-old females who are shorter than 35 inches according to the normal model is close to the actual results. The normal curve accurately models the heights.

Because the area under the normal curve represents a proportion, we can also use the area to find percentile ranks of scores.

Objective 1, Page 7

**Example 2 *Finding and Interpreting Area Under a Normal Curve***

A pediatrician obtains the heights of her 200 three-year-old female patients. The heights are approximately normally distributed, with mean 38.72 inches and standard deviation 3.17 inches. Use the normal model to determine the probability that a randomly selected 3-year-old female is between 35 and 40 inches tall, inclusive. That is, find 

Objective 1, Page 10

 *Answer the following after watching the video.*

1. Summarize the methods for finding the area to the left of *x*, the area to the right of *x*, and the area between  and .

**Area to the Left of *x***

**Area to the Right of *x***

**Area Between  and **

#### Objective 2: Find the Value of a Normal Random Variable

Objective 2, Page 1

Often, we do not want to find the proportion, probability, or percentile given a value of a normal random variable. Rather, we want to find the value of a normal random variable that corresponds to a certain proportion, probability, or percentile. For example, we might want to know the height of a 3-year-old girl who is at the 20th percentile. Or we might want to know the scores on a standardized exam that separate the middle 90% of scores from the bottom and top 5%.

Objective 2, Page 2

**Example 3 *Finding the Value of a Normal Random Variable***

The heights of a pediatrician's 3-year-old female patients are approximately normally distributed, with mean 38.72 inches and standard deviation 3.17 inches. Find the height of a 3-year-old female at the 20th percentile.

Objective 2, Page 4

**Example 4 *Finding the Value of a Normal Random Variable***

The scores earned on the mathematics portion of the SAT, a college entrance exam, are approximately normally distributed with mean 516 and standard deviation 116. What scores separate the middle 90% of test takers from the bottom and top 5%? In other words, find the 5th and 95th percentiles.

Data from The College Board

Objective 2, Page 6

1. What does the notation  represent?

Objective 2, Page 7

**Example 5 *Finding the Value of ***

Find the value of 

Objective 2, Page 9

1. For any continuous random variable, what is the probability of observing a specific value of the random variable?

Since the probability of observing a specific value of a continuous random variable is 0, the following probabilities are equivalent.



## Section 7.3 Assessing Normality

### Objective

1. Use Normal Probability Plots to Assess Normality

#### Objective 1: Use Normal Probability Plots to Assess Normality

Objective 1, Page 1

1. What is a normal score?
2. What is a normal probability plot?
3. List the four steps for drawing a normal probability plot by hand.

Objective 1, Page 4

The idea behind finding the expected z-score is, if the data come from a normally distributed population, we could predict the area to the left of each data value.

Objective 1, Page 5

1. If sample data are taken from a population that is normally distributed, how will the normal probability plot appear?

Objective 1, Page 5(continued)

1. Explain how to determine if a normal probability plot is “linear enough”.

Objective 1, Page 6

**Example 1 *Drawing a Normal Probability Plot by Hand***

The data in Table 2 represent the finishing time (in seconds) for six randomly selected races of a greyhound named Barbies Bomber in the 5/16-mile race at Greyhound Park in Dubuque, Iowa. Is there evidence to support the belief that the variable “finishing time” is normally distributed?

**Table 2**

|  |  |
| --- | --- |
| 31.35 | 32.52 |
| 32.06 | 31.26 |
| 31.91 | 32.37 |

Data from Greyhound Park, Dubuque, IA

Objective 1, Page 8

Typically, normal probability plots are drawn using either a graphing calculator with advanced statistical features or statistical software such as StatCrunch.

Objective 1, Page 9

**Example 2 *Drawing a Normal Probability Plot Using Technology***

Draw a normal probability plot of the Barbies Bomber data in Table 2 using technology. Is there evidence to support the belief that the variable “finishing time” is normally distributed?

**Table 2**

|  |  |
| --- | --- |
| 31.35 | 32.52 |
| 32.06 | 31.26 |
| 31.91 | 32.37 |

Data from Greyhound Park, Dubuque, IA

Objective 1, Page 10

**Example 3 *Assessing Normality***

The data in Table 4 represent the time 100 randomly selected riders spent waiting in line (in minutes) for the Demon Roller Coaster. Is the random variable “wait time” normally distributed?

**Table 4**

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 7 | 3 | 5 | 107 | 8 | 37 | 16 | 41 | 7 | 25 | 22 | 19 | 1 | 40 | 1 | 29 | 93 |
| 33 | 76 | 14 | 8 | 9 | 45 | 15 | 81 | 94 | 10 | 115 | 18 | 0 | 18 | 11 | 60 | 34 |
| 30 | 6 | 21 | 0 | 86 | 6 | 11 | 1 | 1 | 3 | 9 | 79 | 41 | 2 | 9 | 6 | 19 |
| 4 | 3 | 2 | 7 | 18 | 0 | 93 | 68 | 6 | 94 | 16 | 13 | 24 | 6 | 12 | 121 | 30 |
| 35 | 39 | 9 | 15 | 53 | 9 | 47 | 5 | 55 | 64 | 51 | 80 | 26 | 24 | 12 | 0 |  |
| 94 | 18 | 4 | 61 | 38 | 38 | 21 | 61 | 9 | 80 | 18 | 21 | 8 | 14 | 47 | 56 |  |

## Section 7.4 The Normal Approximation to the Binomial Probability Distribution

### Objective

1. Approximate Binomial Probabilities Using the Normal Distribution

#### Objective 1: Approximate Binomial Probabilities Using the Normal Distribution

Introduction, Page 1

 *Answer the following after watching the video.*

1. What are the three criteria for a binomial probability experiment?
2. Under what conditions will a binomial random variable be approximately normally distributed?

Objective 1, Page 1

1. If the binomial random variable *X* is approximately distributed, state the formulas for its mean and standard deviation.

Objective 1, Page 2

1. If *n* = 40 and *p* = 0.5, we can use a normal model because . Compute  and .

Objective 1, Page 4

To approximate the probability of a specific value of the binomial random variable, such as P(18), find the area under the normal curve from x = 17.5 to x = 18.5. We add and subtract 0.5 from x = 18 as a correction for continuity because we are using a continuous density function to approximate a discrete probability.

Objective 1, Page 5

 *Watch the video that summarizes the various corrections for continuity.*

Objective 1, Page 6

1. What is the continuity correction in each of the following cases?
2. *P*(*X* = *a*)
3. 
4. 
5. 

Objective 1, Page 9

**Example 1 *The Normal Approximation to a Binomial Random Variable***

According to the American Red Cross, 7% of people in the United States have blood type O-negative. What is the probability that in a simple random sample of 500 people in the United States fewer than 30 have blood type O-negative?

Objective 1, Page 10

Note that the approximate result using the normal model is only off by 0.0007 from the exact probability computed using technology. Also, notice the shape of the distribution in the StatCrunch output.

Objective 1, Page 12

**Example 2 *A Normal Approximation to the Binomial***

According to the Gallup Organization, 65% of adult Americans are in favor of the death penalty for individuals convicted of murder. Erica selects a random sample of 1000 adult Americans in Will County, Illinois, and finds that 630 of them are in favor of the death penalty for individuals convicted of murder.

* 1. According to the Gallup Organization, 65% of adult Americans are in favor of the death penalty for individuals convicted of murder. Erica selects a random sample of 1000 adult Americans in Will County, Illinois, and finds that 630 of them are in favor of the death penalty for individuals convicted of murder.
  2. Does the result from part (A) contradict the Gallup Organization's findings? Explain.